

Assessment of Concrete Footpath Upheavals in Heatwaves

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Abstract

Most concrete footpaths are cast without steel reinforcement and in segments between bituminous joints, with the joints intended to accommodate thermal expansions of the pavement. However, a combination of prolonged heatwave conditions and the ingress over time of material into the joints produces a situation in which the thermal movements are not in the plane of the pavement, but instead the pavement experiences an upheaval buckle. Within a framework of global warming and heatwaves, assessments of the potential for such buckles which pose a risk to safety are essential. This paper presents a solution in closed form for the combination of key parameters at buckling needed to undertake such an assessment.

Keywords: Upheaval buckling; pavement; non-linear

1. Introduction

The vulnerability of concrete footpaths to buckle in an upheaval mode is increasing with the frequency of long heatwave conditions (Figure 1) and, because such buckles pose a risk to pedestrians and to other traffic, an assessment of their likelihood is needed. Bituminous joints provide for control of shrinkage and for thermal expansion, with dowels often used to transfer shear forces to prevent deflection irregularities. However, the ingress of debris and other matter may prevent the free movement under thermal loading, and the pavement will deflect into the configuration shown in Figures 1



Figure 1. Pavement upheaval (source: author)

2. Non-Linear Analysis

Under uniform loading, the pavement is subjected to a non-mechanical strain $\varepsilon_{th} = \alpha T$, in which α is the coefficient of thermal expansion and T the temperature increment from the so-called “neutral temperature”, corresponding closely to the temperature at which the concrete is cast *in situ*. For a rigid pavement in a Lagrangian (x, y) coordinate system, the total strain can be taken as $\varepsilon = u'$; the prime denoting $d(\)/dx$. In accordance with the Neumann-Duhamel relationship, the total strain is the sum of the stress-producing strain σ/E and thermal strain, so that

$$\sigma = E(u' - \alpha T); \quad (1)$$

E being the elastic modulus. Since $v = x\theta$ (Figure 2) and the non-linear strain is $\varepsilon = u' + \frac{1}{2}v'^2$ (Bradford 2013),

$$\varepsilon = u' + \frac{1}{2}\theta^2. \quad (2)$$

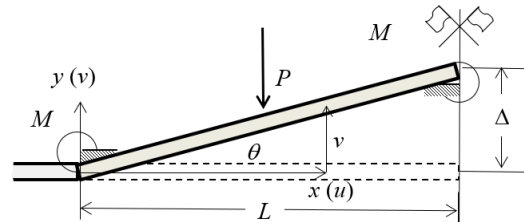


Figure 2. Buckled configuration of jointed pavement.

By definition, the axial force in the pavement is

$$N = \int_A \sigma dA = AE \left(u' + \frac{1}{2}\theta^2 - \alpha T \right). \quad (3)$$

The equations of equilibrium can be derived conveniently by considering the virtual work caused by imparting admissible and arbitrary perturbations δu and $\delta\theta$ to the equilibrium configuration of the pavement. The increase of the internal virtual work is

$$\delta U = \int_0^L \int_A \sigma \cdot \delta\varepsilon dA dx = \int_0^L N (\delta u' + \theta\delta\theta) dx, \quad (4)$$

which produces

$$\delta U = N\delta u \Big|_0^L - \int_0^L N' \delta u dx + NL\theta\delta\theta \quad (5)$$

on integrating by parts. The rotational restraint provided by the dowels crossing the joint is $M = k\theta$, so that the

decrease of potential at each end is $-k\delta\theta$ because the moment opposes the direction of rotation θ , while the decrease of potential of the load P is $-P\delta\Delta/2 = -PL\delta\theta/2$ for the same reason, so that the decrease in potential is

$$\delta V = -\left(\frac{PL}{2} + 2k\right)\delta\theta. \quad (6)$$

The principle of virtual work requires that $\delta U = \delta V$ for all kinematically-admissible arbitrary perturbations δu and $\delta\theta$ of the displacement field, that is

$$N\delta u\Big|_0^L - \int_0^L N'\delta u dx + \left(NL\theta + \frac{PL}{2} + 2k\right)\delta\theta = 0, \quad (7)$$

leading to the equation of equilibrium

$$NL\theta + \frac{PL}{2} + 2k = 0 \quad (8)$$

in the transverse direction and to $N' = 0$, and therefore $u'' = 0$, in the axial direction. The kinematic boundary conditions are $u(0) = u(L) = 0$ (otherwise the pavement segments would separate) and so $u \equiv 0$, and the non-linear equation for upheaval buckling is

$$\theta^3 - 2\alpha T\theta + \eta = 0, \quad (9)$$

in which the term

$$\eta = \frac{P}{AE} \left(1 + \frac{4k}{PL}\right) \quad (10)$$

collects the effects of the self-weight P and dowel bending stiffness k on resisting the uplift buckling. Because $\eta > 0$, when the temperature $T = 0$, the resulting equation $\theta^3 + \eta = 0$ does not have a positive solution for θ and so buckling does not occur. The relationship between the thermal strain αT and buckling deformation θ can also be written as

$$\alpha T = \frac{\theta^2}{2} + \frac{\eta}{2\theta}. \quad (11)$$

Figure 3 plots the buckling strain αT against θ , from which it can be seen that there is a local minimum when $\partial(\alpha T)/\partial\theta = \theta - \eta/(2\theta^2) = 0$, that is when $\theta_0 = (\eta/2)^{1/3}$, and for which

$$(\alpha T)_0 = \frac{3}{2} \left(\frac{\eta}{2}\right)^{2/3} \approx 0.9449\eta^{2/3} \quad (12)$$

defines the critical temperature.

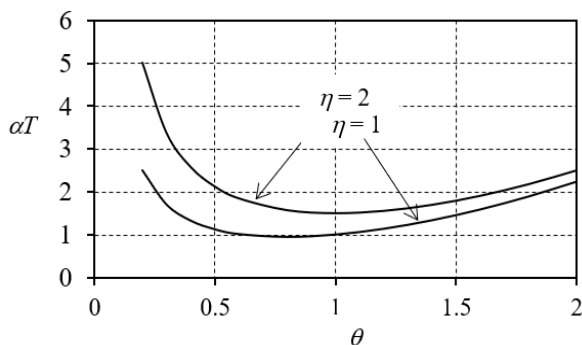


Figure 3. Non-linear upheaval solution

3. Results and Discussion

The solution of Section 2 was applied to a realistic event (CBS Chicago 2012) as shown in Figure 4. For a representative jointed pavement of $L = 6$ m, thickness $2h = 200$ mm, $A = 200 \times 10^3$ mm², $E = 25 \times 10^3$ N/mm², $\gamma = 25$ kN/m³, $\alpha = 10 \times 10^{-6}$ per °C.

$$P = 6 \times 0.2 \times 1.0 \times 25 \text{ kN} = 30 \text{ kN}.$$

Initially, without dowel joints $k = 0$, and hence $\eta = 30 \times 10^3 / (200 \times 10^3 \times 25 \times 10^3) = 6 \times 10^{-6}$; $\theta_0 = (6 \times 10^{-6} / 2)^{1/3} = 14.42 \times 10^{-3}$ radians; $\Delta = 6000 \times 14.42 \times 10^{-3} = 87$ mm; $(\alpha T)_0 = 0.9449 \times (6 \times 10^{-6})^{2/3} = 312 \times 10^{-6}$; and so $T_0 = 312 \times 10^{-6} / (10 \times 10^{-6}) = 31.2$ °C, viz the safe temperature is 31 °C.



Figure 4. Buckled pavement (CBS Chicago 2012)

With the provision of, for example, four dowels of 15 mm diameter per metre of pavement, $I_s = 4 \times \pi \times 15^4 / 32 = 19.88 \times 10^3$ mm⁴; $E_s = 200 \times 10^3$ N/mm²; $k = 2 \times 200 \times 10^3 \times 19.88 \times 10^3 / 100 = 79.52 \times 10^6$ Nmm; $\eta = 6 \times 10^{-6} (1 + 4 \times 79.52 \times 10^6 / (30 \times 10^3 \times 6,000)) = 2.767 \times 16.6 \times 10^{-6}$; $\theta_0 = (16.6 \times 10^{-6} / 2)^{1/3} = 20.26 \times 10^{-3}$ radians; $\Delta = 6000 \times 20.26 \times 10^{-3} = 122$ mm; $(\alpha T)_0 = 0.9449 \times (16.6 \times 10^{-6})^{2/3} = 614.9 \times 10^{-6}$; $T_0 = 614.9 \times 10^{-6} / (10 \times 10^{-6}) = 61.5$ °C. These dowels therefore raise the safe temperature by 30 °C.

4. Conclusions

This paper has presented a formulation to describe the upheaval buckling of a jointed concrete pavement in a closed form solution that captures the influence of the parameters that describe the buckling. It was applied for the purposes of demonstration to a situation reported in the literature, showing close agreement for the buckling temperature reported and that produced from the analysis of the buckling mechanism. It provides a basis for the vulnerability assessment of pavements in circumstances of the need for climate adaption and mitigation.

References

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