Fuzzy Relation between the RDI_{st} Index and the Water Table of a Coastal Aquifer of Nestos Delta, Greece

Papadopoulos Ch.[1,*], Gkıougkıs I.[1], Spiliotis M.[1], Pliakas F.[1], Papadopoulos B.[2]

1Department of Civil Engineering/School of Engineering, Sector of Hydraulics, Democritus University of Thrace, Xanthi, Greece
[2]Department of Civil Engineering/School of Engineering, Sector of Mathematics, Programming and General Courses, Democritus University of Thrace, Xanthi, Greece

*corresponding author: Papadopoulos Christopher: e-mail: cpapadp@civil.duth.gr

Abstract
Drought is a complicated natural extreme event associated with the decline of water availability below the normal conditions of a hydrological system, both from a temporal and from a spatial point of view. This work aims to assess the drought impacts on groundwater fluctuation in a shallow coastal unconfined aquifer via fuzzy approach. Meteorological drought intensity is estimated by the Standardized Reconnaissance Drought Index (RDI_{st}) based on precipitation (P) and potential evapotranspiration (PET). In addition, groundwater modeling is carried out using MODFLOW and then, the simulated values of water table (WT) of a coastal unconfined aquifer are utilized. In order to relate the RDI_{st} with WT, a fuzzy linear regression (FLR) is applied. FLR based on Tanaka model produces a fuzzy band, where all the data must be included within, incorporating the system uncertainty. The suitability of the achieved fuzzy regression model is tested by using appropriate measures. The propounded methodology is applied in the eastern area of Nestos River Delta, Prefecture of Xanthi, Greece.
Keywords: Reconnaissance Drought Index, coastal aquifers, groundwater modeling, fuzzy linear regression, Nestos River Delta.

1. Introduction

Drought can occur both in areas with significant rainfall and in areas with low rainfall and virtually under all climatic regimes (Spiliotis et al. 2016). The assessment of drought impacts on underground reservoirs is of great importance, since the groundwater may act as a natural storage and defense against water shortages during the periods of drought. The recovery of these components is slow because of long recharge periods for subsurface water supplies (Wilhite et al., 2014), although, in the case of a coastal unconfined aquifer, it would be interesting to look at its short-term response to meteorological drought.

Drought intensity is commonly evaluated through the use of drought indices. In this work, the Standardized Reconnaissance Drought Index (RDI_{st}), which is generally suitable for dry and semi-dry areas, is used. The RDI_{st} is computed for four reference periods within the year, for the hydrological years of 2008 and 2009, while potential evapotranspiration is calculated by the Thornthwaite method. Subsequently, the RDI_{st} of each reference period is correlated with log-transformed values of groundwater level of a coastal unconfined aquifer, in the eastern part of the Nestos River Delta, Prefecture of Xanthi, Greece. The water level data derived by using MODFLOW, based on primary data from five wells.

Recently, research works have been carried out utilizing the RDI index and hydrogeological variables in an attempt to evaluate the drought effects both from a quantitative and qualitative perspective on groundwater resources (Nohegar and Heydarzadeh, 2016, Touhidul Mustafa et al. 2017, Vangelis et al., 2017). In this work, a fuzzy linear regression (FLR) methodology is applied, in order to relate the RDI_{st} values and the log-transformed water table values of a coastal unconfined aquifer. FLR based on Tanaka model (1987) leads to a constrained optimization problem producing a fuzzy band where all the data must be included within. Regression coefficients are considered as symmetric triangular fuzzy numbers incorporating the system uncertainty which derived of the hydrological cycle complexity.

2. Case Study

2.1. Materials and methods

The RDI_{st} is computed by the following equation:

$$RDI_{st(\mathrm{k})} = \frac{y_{(\mathrm{i,k})} - \bar{y}_{(k)}}{\hat{\sigma}_{(k)}}, \quad i = 1,2, \; k = 1,..,4 \qquad (1)$$

in which $y_{(\mathrm{i,k})}$ is the $\ln\left(a_{0(i,k)}\right)$ of i^{th} hydrological year for k^{th} reference period within the hydrological year, $\bar{y}_{(k)}$ is its arithmetic mean and $\hat{\sigma}_{(k)}$ is its standard deviation of each reference period (Tsakiris et al., 2007), and where:

$$a_{0(\mathrm{i,k})} = \frac{\sum_{j=1}^{4k} P_{(\mathrm{i},j)}}{\sum_{j=1}^{4k} PET_{(i,j)}}, \quad k = 1,2,3,4 \qquad (2)$$

in which j is the month of each reference period k, k = 1 for October-December, k = 2 for October–March, k = 3 for October–June, and k = 4 for October–September. For the implementation of Tanaka model, symmetric triangular fuzzy numbers (Figure 1a) are adopted:

$$WT_{(i,k)} = \tilde{A}_0 + \tilde{A}_1 RDI_{st(k)}, \quad k = 1,2,3,4 \tag{3}$$

in which $WT_{(i,k)}$ is the water table value derived by using MODFLOW for the months December-March-June-August of the i^{th} hydrological year. According to the Tanaka method (1987), the total spread (fuzziness), J, of the fuzzy output, WT, is minimized under the inclusion constraints. In other words, all the water table values must be included within the produced fuzzy band.

$$J = \min\left\{ mc_0 + \sum_{j=1}^{m}\left(c_1 \left|RDI_{st(k)}\right|\right)\right\} \quad m = 1,2,...,8$$
$$c_0, c_1 \geq 0 \tag{4}$$

where m is the number of the $RDI_{st(k)}$ of the two hydrological years. The decision variables of the equivalent optimization problem are the centres (r_0, r_1) and the widths (c_0, c_1) of the fuzzy regression coefficients A_0 and A_1, correspondingly.

The applicability of the fuzzy regression model can be tested by examining of the magnitude of fuzziness, J, (Papadopoulos and Sirpi, 1999). Apart from measure J, another suitability measure, M, is applied. The measure M counts the mathematical distance between the centres, r_{WT}, of the water table values estimated by fuzzy regression model and the initial water table values, WT, simulated by using MODFLOW. The lower M value the more applicable the regression model is. M is calculated by the following equation:

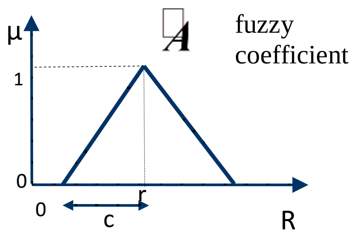
$$M = \sqrt{\left(\sum_{1}^{8}\left(r_{WT} - WT\right)^2\right)} \tag{5}$$

2.2. Results and concluding remarks

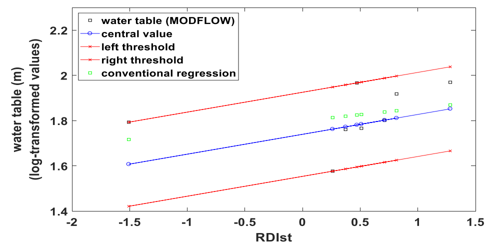
The results show (Table 1, Figure 1b) that the values of both two suitability measures (J, M) are acceptable and interpretable. In particular, in the case of well III and well V the results are improved. It is worth noting that the regression coefficient A_1 is crisp number in all cases, since its width (c_1) takes the zero value. Last, the low M values imply a satisfactorily model applicability in case of data insufficiency.

Table 1. The coefficients of fuzzy and conventional regression as well as the values of suitability measures.

		Wells	I	II	III	IV	V
Regression coefficients	Fuzzy	r_0	1.893	1.769	1.770	1.695	1.740
		c_0	0.235	0.241	0.198	0.236	0.186
		r_1	0.059	0.101	0.098	0.113	0.088
		c_1	**0**	**0**	**0**	**0**	**0**
	Crisp	A_0	1.977	1.855	1.843	1.774	1.799
		A_1	0.059	0.050	0.059	0.066	0.055
Suitability measures		J	1.884	1.931	1.582	1.886	**1.487**
		M	0.458	0.464	0.389	0.448	**0.360**



a)



b)

Figure 1. a) Graphical representation of symmetric triangular fuzzy number, b) FLR applied in the case of well V.

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